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HIGH-SPEED GAS MOTION IN A POROUS MEDIUM

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The problems of motion of a gas in a porous medium have been solved repeatedly, starting with the work of L. S. Leibenzon, and mainly relating to filtration of gas in beds. Reference [1] has reviewed this topic. As a rule, the Darcy resistance law is used, valid for small flow velocities. Even in this formulation the gas compressibility leads to nonlinearity. Therefore, very few exact solutions of unsteady problems have been obtained, and mainly to similarity problems [2-4].

There is presently an interest in high-speed gas flow, associated with the development of investigations of two-phase reacting systems. In two-phase detonation or fast convective combustion [5, 6], the relative speed of the gas and the particles can reach several hundred meters per second. To understand these processes and monitor numerical solutions in their modeling it is desirable to have accurate solutions of the unsteady equations. For the problem of expulsion of a gas from a porous medium the author has obtained asymptotic solutions describing the flow at sufficiently large time values.

1. Statement of the Problem. Ahead of the combustion front in a two-phase system there is a flow region with no chemical reaction (the filtration zone or the air plug). The friction between the gas and the particles in this region is overcome by the dynamic head of fresh combustion products.

We turn now to the following statement of the problem. A "liquid" piston, permeable for particles and impermeable for the gas, is moving according to a given law in a porous medium. We require to find the motion of the gas ahead of the piston.

We assume that there is negligible motion of the particles because of the strength of the solid or the high density. The Darcy law does not hold for high speed motion. In the free charge of particles of diameter 1 mm and flow velocity 100 m/sec the Reynolds number based on diameter is on the order of 10^4. Therefore, the main contribution to the interphase interaction does not come from the viscosity, but from the inertia of fine-scale gas flows. The real resistance law is quadratic, and we shall write it in the form $f = A\Phi u^2/d$, where u is the gas velocity; Φ is the porosity; d is the particle diameter; and A is a coefficient on the order of 1, depending on the porosity and the structure of the void space.

Since the particles are at rest, the porosity is constant. The basic equations for the gas have the form

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$$\rho_t + (\rho u)_x = 0,\ (\rho u)_t + (\rho u^2)_x + \langle \rho\delta u^2\rangle_x + p_x = -\rho f/\Phi,$$
$$(\rho(E + u^2/2 + K))_t + (\rho u(E + u^2/2 + K) + pu)_x + W_x = -q/\Phi,$$
$$W = \langle \rho\delta u(\delta I + u\delta u + K)\rangle,\ I = E + p/\rho. \tag{1.1}$$

Here, in analogy with [7], ρ is the density, averaged over a "representative" small gas volume; u, E, I are the bulk quantities; the average surface pressure is p; q is the heat transfer. The equations take account of the fluctuation of momentum transfer $\langle \rho\delta u^2 \rangle$ and the energy K of the fluctuating motion. Usually these fluctuations are neglected in calculations [8, 9]. However, although the volume of the solid phase fraction $(1 - \Phi)$ is not small, the gas moves along very winding channels, and the fluctuations will be on the order of the quantities themselves. Therefore, in the porous medium one must allow for the fluctuations in all cases where one has to calculate the inertia terms and the kinetic energy of the mean motion.

For "smooth" flows where the characteristic scale $L \gg d$ and the characteristic time $T \gg d/u$, the time for flow over a grain, one naturally expects that

$$\langle \rho\delta u^2\rangle = \delta_1\rho u^2,\ K = \delta_2 u^2/2,\ W = \delta_3\rho u^3, \tag{1.2}$$

where the coefficients δ_1, δ_2 and δ_3, which depend on the geometry of the structure and the porosity, are of order 1. In this case one can neglect the inertia terms and the fluctuations:

$$\rho_t + (\rho u)_x = 0,\ p_x = -\rho f/\Phi,\ p_t + \gamma(pu)_x = -(\gamma - 1)q/\Phi. \tag{1.3}$$

Equations (1.3) are typical for filtration problems allowing for heat removal. The gas is considered perfect with adiabatic index γ. Here the inequalities

$$p \sim L\rho f/\Phi = A\rho u^2 L/d \gg \rho u^2,\ E = p/(\gamma - 1)\rho \gg u^2/2,\ K$$

are valid.

We move the "liquid" piston with constant velocity v in a medium that is initially at rest. Below we consider two limiting cases: 1) a constant temperature gas (perfect heat removal and large specific heat of the particles); 2) zero heat removal (the heat of interphase friction goes to heating of the gas).

Clearly, the isothermal case corresponds to rather low velocities when heat transfer can occur. This approximation is well known in filtration theory. On the other hand, one can neglect heat transfer at high velocities.

2. Isothermal Problem. At constant temperature $p = c^2\rho$ (c is the isothermal sound speed). This relation must be used in lieu of the energy equation. For comparatively small velocities it makes sense to consider a two-term resistance law:

$$f/\Phi = -Au^2/d - bu.$$

At the piston (for $x = vt$) $u = v$, $\rho(x, 0) = \rho(\infty, t) = \rho_0$, $u(\infty, t) = 0$. We shall seek ρ and u in the form $\rho = \rho_0 R(\xi, \tau)$, $u = vU(\xi, \tau)$, where $\xi = (x - vt)/L$ is the dimensionless coordinate in the piston system; $\tau = vt/L$ is the dimensionless time; and $L = (c/v)^2 d/A$ is the characteristic scale of the flow. From system (1.3) we obtain

$$R_\tau - R_\xi + (RU)_\xi = 0,\ R_\xi/R = -U(U + \alpha),$$

where $\alpha = bd/Av$. Eliminating R we have the equation for the velocity:

$$\frac{\partial U}{\partial \xi} = -U(U + \alpha)(1 - U) - \frac{\partial}{\partial \tau}\int_\xi^\infty U(U + \alpha)\,d\xi. \tag{2.1}$$

We are interested in the solution for large values of time τ. Of course, here we cannot obtain strictly steady-state conditions. If we omit the dependence on time, then at the piston for $U = 1$ we have $\partial U/\partial\xi = 0$ and $U \equiv 1$. We can obtain the steady-state solution if we impose the condition $U = 1$, not at the piston ($\xi = 0$), but for $\xi = -\infty$. This formulation was considered in [4].

However, in some sense the problem has almost a steady-state solution, in which the time dependence is appreciable only for $U \approx 1$, i.e., near the piston. We shall postulate that the equality

$$\frac{\partial}{\partial \tau}\int_\xi^\infty U(U + \alpha)\,d\xi = BU(U + \alpha)/\tau \tag{2.2}$$

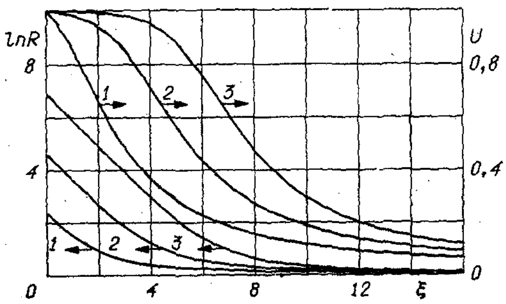

Fig. 1

holds, with an accuracy up to small quantities $\sim\tau^{-2}$. Here B is a constant coefficient as yet unknown. Taking account of Eq. (2.2) we can easily integrate Eq. (2.1):

$$\xi=\frac{\ln\left(\frac{B+\tau-U\tau}{B}\right)}{(1+B/\tau)(1+\alpha+B/\tau)}+\frac{\ln\left(\frac{U+\alpha}{1+\alpha}\right)}{\alpha(1+\alpha+B/\tau)}-\frac{\ln U}{\alpha(1+B/\tau)}.$$

For $\alpha = 0$ (purely quadratic resistance law)

$$\xi=\ln\left(\frac{B+\tau-U\tau}{BU}\right)\Big/(1+B/\tau)^2+(1/U-1)/(1+B/\tau).$$

Using these solutions we obtain

$$\int\limits_{\xi}^{\infty} U(U+\alpha)\,d\xi=\ln\left(\frac{B+\tau}{B+\tau-U\tau}\right).$$

The density is expressed by the formula

$$R=\frac{B+\tau}{B+\tau-U\tau},\quad R(\xi=0,\tau)=1+\tau/B.$$

A check that the hypothesis of Eq. (2.2) holds in the solution obtained gives

$$\frac{\partial}{\partial\tau}\int\limits_{\xi}^{\infty} U(U+\alpha)\,d\xi=\frac{U(U+\alpha)}{\tau(1+\alpha)}+O(\tau^{-2}),$$

whence $B = 1/(1+\alpha)$. The same result is obtained from the condition that the mass of gas undergoing combustion is conserved:

$$\int\limits_{0}^{\infty}(R-1)\,d\xi=\tau$$

with an accuracy up to terms of $O(1)$.

Figure 1 shows the profiles of U and R for $\alpha = 0.01$ (a small contribution of the linear resistance) for the times $\tau = 10$; 10^2; 10^3 (curves 1-3, respectively). As time goes on the profile of velocity, while basically retaining its shape, moves "logarithmically" ahead, so that the size of the perturbed region is close to $\ln\tau$. In this region the density falls off exponentially with the coordinate, and increases proportionally with time.

Inclusion of the inertial terms in the momentum equation leads to the appearance of discontinuities if the wave speed exceeds c. It is dubious that the isothermal feature can be preserved under these conditions, and we shall not analyze this case in detail.

3. Zero Heat Removal. For system (1.1) we seek a solution in the form of a wave in which ρ = const, $u = v$. We shall postulate that the fluctuation terms are functions of the mean velocity u and the density ρ, e.g., in accordance with Eq. (1.2). The first equation holds automatically, and the system is simplified: $p_x = -\rho A v^2/d$, $p_t + \gamma v p_x = 0$.

It can be seen that the wave front velocity is $D = \gamma v$ and that the dependence of $p(x)$ is linear. The density and pressure in the wave are expressed by the formulas

$$\rho=\frac{\gamma}{\gamma-1}\rho_0,\quad p=\frac{\gamma}{\gamma-1}\rho_0 A v^2\frac{\gamma vt-x}{d}+p_0. \tag{3.1}$$

A solution exists in the region between the piston $x = vt$ and the wave front $x = \gamma vt$. The maximum pressure at the piston grows linearly with time. A continuation of the formulas (3.1) to $x = 0$ gives a solution of the problem of blowing of gas into a semispace at constant rate. We can obtain the steady-state solution if the piston moves with velocity γv and is partially permeable for the gas.

The solution (3.1) is applicable for sufficiently long waves ($vt \gg d$). Near the front there is a transition region of width on the order of the particle diameter in which there is a flow adjustment from a state of rest to the steady solution (3.1). For a subsonic wave front velocity the transition is smooth, and a discontinuity arises for supersonic velocity. A detailed investigation of this situation is inappropriate until one establishes relations for the fluctuations in thezone of abrupt flow variations. In the same region we develop the constant term $p_0 \sim \rho v^2$, which can be neglected for long waves.

The results of this section with the obvious changes hold for any resistance law of the form $f(\rho, u)$.

4. Application of the Solutions Obtained. The isothermal problem may be of interest for classical filtration applications (expulsion of a gas by a liquid).

We can neglect heat removal from the gas under the condition $p_t \gg (\gamma - 1)q/\Phi$.

Using Eq. (3.1) we can show that the process time t must be much less than the largest of the two characteristic times:

$$t_1 = (\mathrm{Re}\ \mathrm{Pr}/\mathrm{Nu})(d/v),\ t_2 = (\rho C/\rho_s C_s)^2(d^2/\chi_s),$$

where Nu is the Nusselt number; ρ_s, C_s, χ_s are the density, specific heat and thermal diffusivity of the particles. Estimates show that for $d = 1$ mm, $v = 1$ km/sec t_1 and t_2 are roughly 30 μsec, i.e., the heat transfer becomes appreciable when the wave passes over several tens of grains. Thus, the adiabatic solution (3.1) is limited with respect to time both below (time to flow over the particle) and above (time for heat transfer).

In an experiment [6] a "precursor" was observed with a linear pressure increase for a low-speed ($D \approx 1$ km/sec) two-phase detonation. It is possible that this zone is an air slug which pushes a "piston" of reaction products. This explanation agrees well with the experiment if the friction coefficient A is close to 1.

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